Buckminster Primary School's

Progression through

Written Calculation

This document aims to exemplify the standards expected for children on their journey through calculation. Strategies included in this document are all NCETM and DfE approved strategies and meet the aims and programmes of study set out in the National Curriculum 2014. It is vital that each child, as stated in the National Curriculum 2014, has their learning personalised to their appropriate stage of understanding; not their age. When children conceptually understand the learning and are able to apply the skills in a variety of contexts, they are ready to move on to the next stage.

> The responsibility of mathematics education is to enable all pupils to develop conceptual understanding of the mathematics they learn, its structures and relationships, and fluent recall of mathematical knowledge and skills to equip them to solve familiar problems as well as tackling creatively the more complex and unfamiliar ones that lie ahead.
>
> While the best teaching developed pupils' conceptual understanding alongside their fluent recall of knowledge, and confidence in problem solving, too much teaching concentrated on the acquisition of disparate skills that enabled pupils to pass tests and examinations but did not equip them for the next stage of education, work and life.
>
> Being 'made to measure' might describe schools' perceptions of, and reaction to, the pressures to raise standards. However, the aim for all schools should be to secure high calibre, 'made-to-measure' mathematics provision to optimise every pupil's chance of the best mathematics education.
>
> (Mathematics: Made to Measure, OfSTED, May 2012)

It is important that all members of staff, regardless of their career stage, work together to understand and implement the key messages of this document in order to ensure that teaching approaches ensure good progress for all pupils. Senior leaders have a responsibility to provide professional development for teachers and teaching assistants to ensure they meet the new Teachers' Standards to ensure they understand the developing progression and see how the methods in any year build on what went before and feed into what is learned later.

> A teacher must:
>
> - Have a secure knowledge of the relevant subject(s) and curriculum areas, foster and maintain pupil's interest in the subject and address misunderstanding;
> - Demonstrate a critical understanding of developments in the subject and curriculum areas and promote the value of scholarship;
> - If teaching early mathematics, demonstrate a clear understanding of appropriate teaching techniques
>
> - (Teachers' Standards, DfE, May 2012)

Glossary of terms

Cardinal number	The number of items in a set, the quantity but not the order of things. For example, 'There are five pencils in a pot.'
Conservation of number	If a group of objects is rearranged, the total number of objects stays the same.
Consecutive	Following in order. Consecutive numbers are adjacent in a count. For example, 5, 6, 7 are consecutive numbers. 25, 30, 35 are consecutive multiples of 5.
Commutativity	For addition and multiplication, the numbers in a calculation can be in any order and will result in the same answer. E.g. 3x4=12 and 4x3=12 or 3+4=7 and 4+3=7. Addition and multiplication are commutative. Subtraction and division are not commutative. However, children must understand that the numbers in a calculation can also be in any order but will result in a different answer. E.g. 7-5=2 and 5-7=-2.
Digit	One of the symbols of a number system, most commonly the symbols 0,1, 2, 2, 4, 5,6, 7, 8 and 9. For example, the number 29 is a two-digit number; 5 is a one-digit number. The position or place of a digit in a number conveys its value.
Dividend	The quantity which is to be divided. E.g. in the calculation 12 ÷ 3, the dividend is 12.
Divisor	The quantity by which another quantity is to be divided. E.g. for the calculation 12 ÷ 3, the divisor is 3.
Estimate	Verb: To arrive at a rough approximate or answer. Noun: A rough approximate or answer.
Fewer	Used to compare two or more sets of countable (discrete) objects. For example, 'There are fewer biscuits on this plate than on that plate', or 'There are two fewer apples in this bag.'
Less	Used to compare 'uncountable' (continuous) quantities including measures. For example, 'This bottle has less water in it than that one.'
Long Multiplication	A formal calculation strategy that builds on understanding of the grid method into a compact column method. The multiplier is larger than 12 and is therefore partitioned during the process to aid calculation. Long multiplication is a multi-stage calculation which requires a final addition calculation in order to reach the final outcome.
Inverse of multiplication (as a method of division)	Counting up from 0 in multiples to reach a number in order to solve a division calculation. Inverse of multiplication is used to see how many amounts make a given number. E.g. starting at 0 and counting up in steps of 3 until 12 is reached. Some children find counting on in the multiples from 0 easier than repeated subtraction and this is fine so long as they understand they are using the inverse of multiplication rather than repeated subtraction.
Number line	A line on which numbers are represented by points. Division marks are numbered, rather than spaces. The can begin at any number and extend into negative numbers. They can show any number sequence. 012345678910
Number track	A numbered track along which counters may be moved. The number in a region represents the number of single moves from the start. • Each number occupies a cell and is used to number the cell. • Numbers may have a matching illustration • Supports learning to read numbers in numerals. • Supports locating ordered numbers. • They should start at 1 and not 0.

Glossary of terms (continued)

Numeral	A symbol used to denote a number. For example, 5, 23 and the Roman V are all numbers written in numerals.
Ordinal numbers	A term that describes a position within an ordered set. For example, first, second, third, fourth…twentieth.
Partition	1. To separate into subsets. 2. To split a number into component parts. For example, the two-digit number 38 can be partitioned into 30+8 or 19+19.
Pattern	A systematic arrangement of numbers, shapes or other elements according to a rule.
Principle of Exchange	The naming system when collections, that as soon as we have a group of ten we call them something else. The number we call ten (10 in numerals) is the most important in our naming system. E.g. ten ones are called one ten, ten tens are called one hundred, ten hundreds are called one thousand.
Proportionality	The relationship of one thing to another in terms of quantity, size, or number / out of the whole / 2 out of 5. Proportionality puts the emphasis on the relationship rather than the quantity.
Quotient	The result of a division calculation. E.g. In the calculation of 12 ÷ 3, the quotient is 4.
Ratio	The comparison of two properties / 2:3. All ratio relationships are proportional.
Repeated subtraction	Repeatedly subtracting the same amount each time in order to solve a division calculation. The idea of repeated subtraction should be 'how many times can I take ____away from____?' E.g. 12 ÷ 3 using repeated subtraction we should start at 12 and repeatedly count down in steps of 3 until 0 is reached.
Representation	The wide variety of ways to capture an abstract mathematical concept or relationship. This may be visible, such as a number sentence, a display of manipulative materials, or a graph, but it may also be an internal way of seeing and thinking about a mathematical idea. Regardless of their form, representations can enhance students' communication, reasoning and problem-solving abilities; help them make connections among ideas and aid them in learning new concepts and procedures.
Sequence	An ordered set of numbers or shapes arranged according to a rule.
Short Multiplication	A formal calculation strategy that builds on understanding of the grid method into a compact column method. The multiplier is 12 or less and is therefore not partitioned during the process as the calculations should rely on knowledge of key multiplication facts up to 12x12. An expanded short multiplication method details each stage in brackets and shows clear connections to the grid method. This will be used as a vital stage in bridging understanding from the grid method to short multiplication.
Subitising	This is a process whereby we recognise the size of a set, its cardinality, from the pattern or structure without having to count the number of objects. For example, recognising there are five dots in this pattern.
Zero	1. Nought or nothing 2. In a place-value system, a place-holder. For example, 105. 3. The cardinal number of an empty set.

Addition and Subtraction

+

−

Stage 1	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

+ + + + + + + + Addition + + + + + + + +

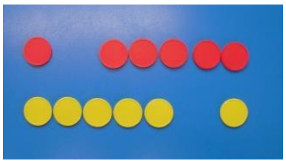
Vocabulary – Ensure the correct vocabulary is used at all stages of learning
add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse, how many more to make…?, is the same as

Children will use practical equipment to combine groups of objects to find a total. Practical resources will support children's development of mental pictures and images.

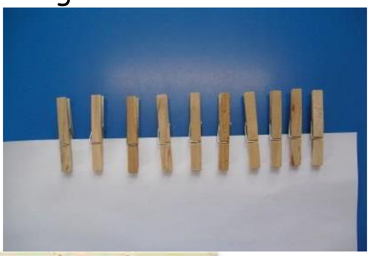
Children will begin to understand commutativity and the principle of exchange. They will be confident in using the terms 'worth' and 'value' when talking about single-digit numbers.

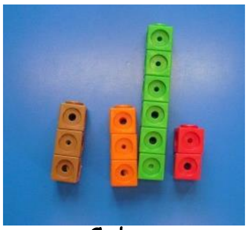
Children can represent calculations using objects and talk about their representations.

Fingers



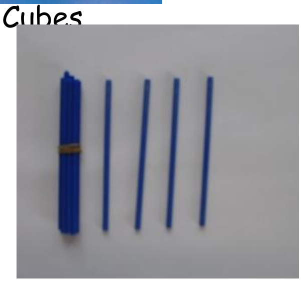
Beads/counters or any object





Cubes

Cuisenaire Rods



Straws

Stage 1	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

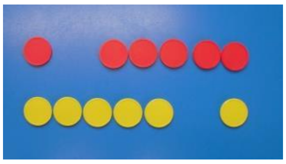
- - - - - - - - Subtraction - - - - - - - -

Vocabulary – Ensure the correct vocabulary is used at all stages of learning
subtract, subtraction, take away, minus, decrease, leave, how many are left/left over?, difference between, half, halve, how many more/fewer is…/than…?, how much more/less is…?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

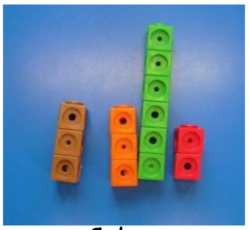
Children will use practical equipment to physically remove an amount from the group to find the total remaining. Practical resources will support children's development of mental pictures and images.

Children can represent calculations using objects and talk about their representations.

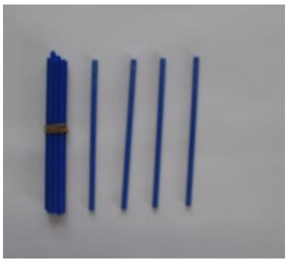

Fingers


Beads/counters or any object

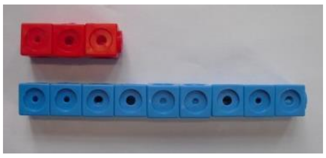

Pegs


Cubes


Cuisenaire Rods


Straws

Children will also be introduced to the language of comparison including equal use of vocabulary 'less' and 'more'.


There are more blue than red.
There are less red than blue.

Stage 2	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

+ + + + + + + + Addition + + + + + + + +

Vocabulary – Ensure the correct vocabulary is used at all stages of learning
add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse, how many more to make…?, is the same as

Practical resources will continue to support children's development of mental pictures and images. As these become firm, children will begin to develop ways to represent their mental images and their practical resources using pictures.

The children will begin to use number sentences alongside their pictures and practical resources.

They will also begin to think flexibly about addition.

The direct link between addition and subtraction should be made explicit when using models and representations.

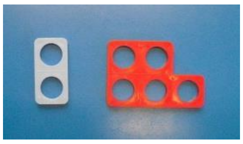

10 = 5 + 5

5 and 1 more is?
5 and 2 more is?
5 and 3 more is?

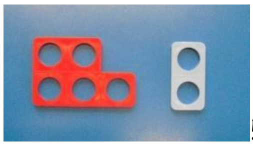
Count on in one, two or three

9 and 1 more is 10
9 add 1 equals 10
9 + 1 = 10


10 = 7 + 3

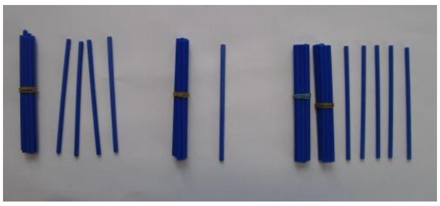

2 + 5 = 7

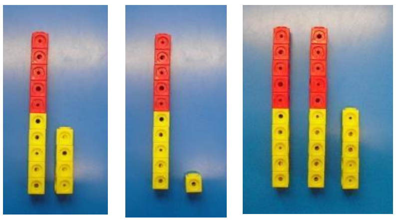

10 = 3 + 7


5 + 2 = 7


10 = 6 + 4   4 + 6 = 10


8 + 2 = 10


14 + 11 = 25


14 + 11 = 25

Stage 2	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

- - - - - - - - Subtraction - - - - - - - -

Vocabulary – Ensure the correct vocabulary is used at all stages of learning
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Practical resources will continue to support children's development of mental pictures and images. As these become firm, children will begin to develop ways to represent their mental images and their practical resources using pictures.

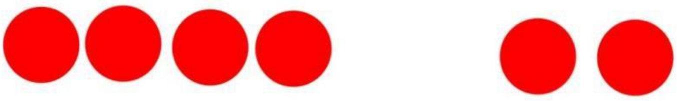
The children will begin to use number sentences alongside their pictures and practical resources.

They will also begin to think flexibly about subtraction and make links to the inverse of addition. Children will understand that subtraction is not commutative and so the numbers in a calculation can be in any order but will result in a different answer.

The direct link between addition and subtraction should be made explicit when using models and representations.

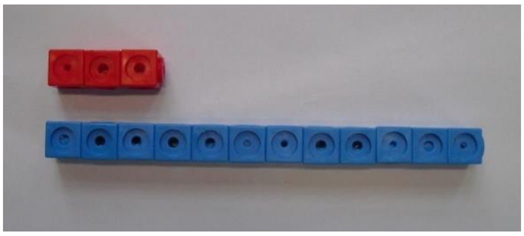
6 + ? = 10 ? + 6 = 10
10 – 6 = ? 10 – 4 = 6

10 – 3 = 7



6 – 2 = 4

Children will continue to be introduced to the language of comparison and its link to finding the difference structure of subtraction.



There are more blue than red.
There are less red than blue.
There are 9 more blue than red.
There are 9 less red than blue.

Stage 3	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

+ + + + + + + + Addition + + + + + + + +

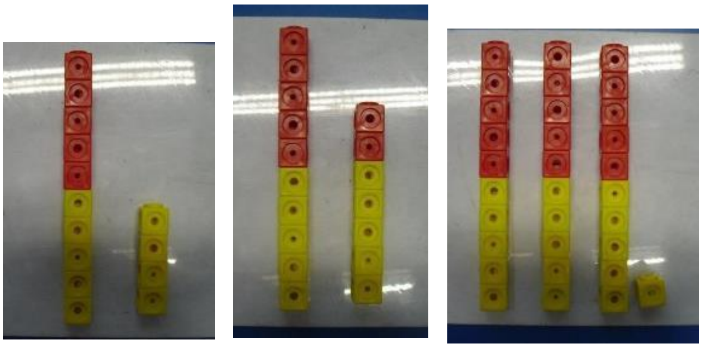
Vocabulary – Ensure the correct vocabulary is used at all stages of learning
add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse, how many more to make…?, is the same as

Children will now be confident in using concrete equipment to help them combine groups of objects with numbers up to 20.

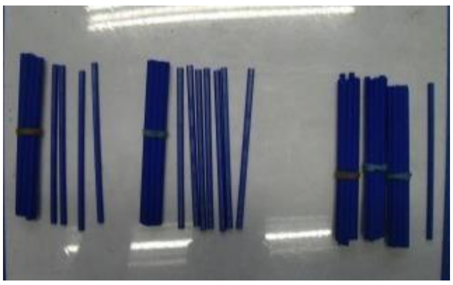
They will continue using practical equipment as they begin to also use number tracks, number lines and hundred squares to support their mental methods.

Children will start to work with totals greater than 20 which require them to apply their knowledge of the principle of exchange. They will talk confidently about this.

14 + 17



14 + 17 = 31



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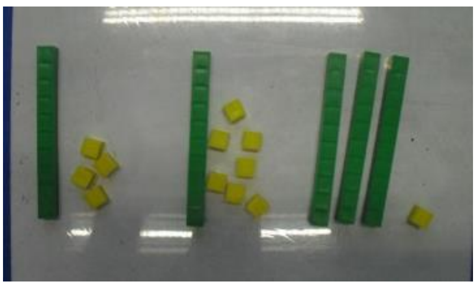

14 + 17 = 31

Stage 3	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

- - - - - - - - Subtraction - - - - - - - -

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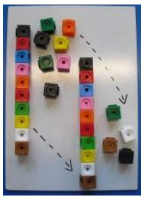
Children will now be confident in using concrete equipment to help them 'take away' and 'find the difference'.

They will continue using practical equipment as they begin to also use number tracks, number lines and hundred squares to support their mental methods.

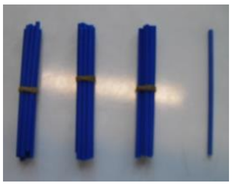
Children will start to work with numbers greater than 20 which require them to apply their knowledge of the principle of exchange. They will talk confidently about this.

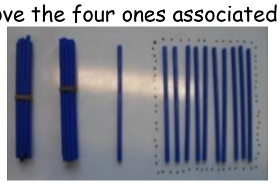
31 - 14

31 is repartitioned into 20 and 11 using the principle of exchange in order to enable us to remove the four ones associated with 14



14 can now be removed from the 31 leaving 17





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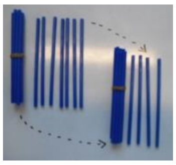

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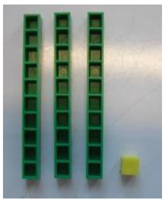

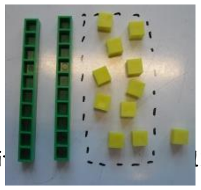

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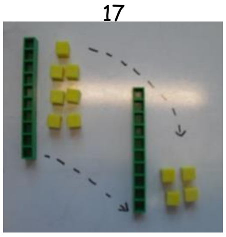
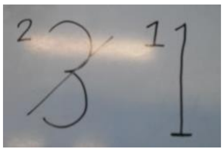


14 can now be removed from the 31 leaving 17

As children become accustomed to repartitioning numbers, they can be introduced to formal notation of the repartitioning.



2 tens + 11 ones = 31

Stage 4

Points to remember
- Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total')
- Use the language '**digit**' not number (number is amount or quantity)

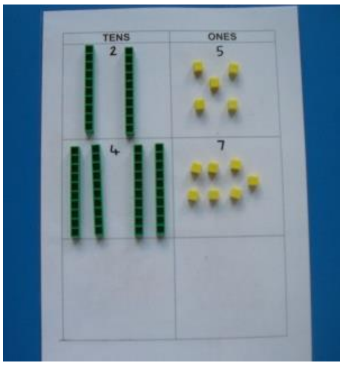
+ + + + + + + + Addition + + + + + + + +

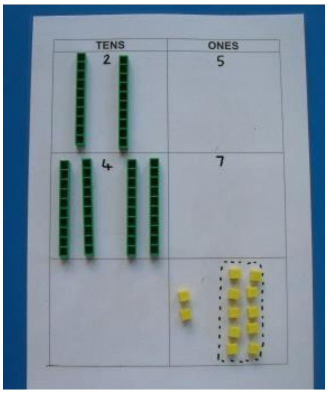
Vocabulary – Ensure the correct vocabulary is used at all stages of learning
add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse, how many more to make…?, is the same as

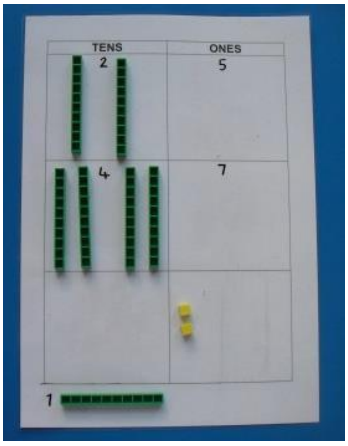
Children are now confident in using concrete equipment to combine objects using the principle of exchange appropriately.

They will now begin to organise their concrete equipment (e.g. Straws, Dienes, Place Value Counters) in a vertical manner where their combined totals are situated at the bottom.

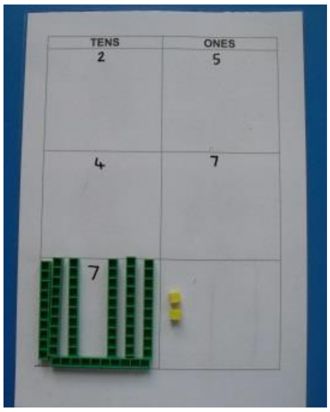
25 + 47







12 ones exchanged to 1 ten and 2 ones



Stage 4	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

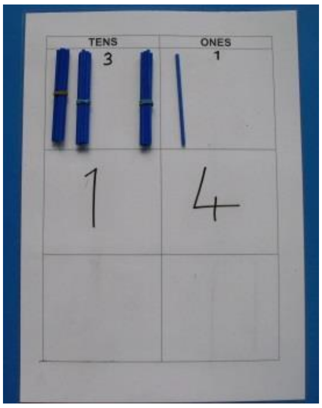
- - - - - - - - Subtraction - - - - - - - -

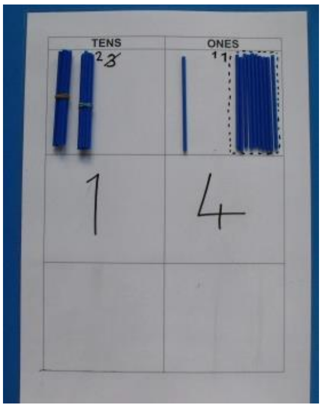
Vocabulary – Ensure the correct vocabulary is used at all stages of learning
subtract, subtraction, take away, minus, decrease, leave, how many are left/left over?, difference between, half, halve, how many more/fewer is…/than…?, how much more/less is…?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children are now confident in using concrete equipment to help them 'take away' and 'find the difference' using the principle of exchange appropriately.

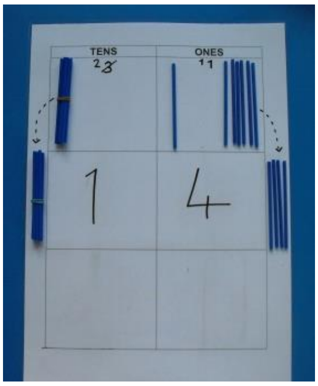
They will now begin to organise their concrete equipment (e.g. Straws, Dienes, Place Value Counters) in a vertical manner where the amount that remains at the end of the calculation is situated at the bottom.

31 - 14

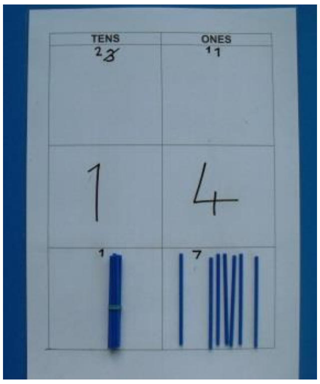




31 is repartitioned into 20 and 11 using the principle of exchange in order to enable us to remove the four ones associated with the 14



14 can now be removed from the 31



The remaining equipment can be slid down to the answer box showing what is left

Stage 5	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

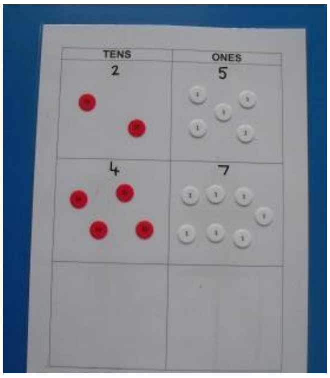
+ + + + + + + + Addition + + + + + + + +

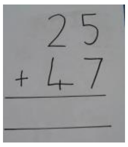
Vocabulary – Ensure the correct vocabulary is used at all stages of learning
add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse, how many more to make…?, is the same as

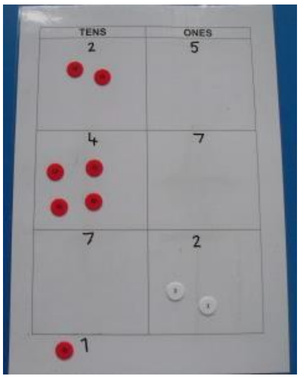
Children will now be secure in organising their concrete equipment in a vertical manner where their combined totals are situated at the bottom.

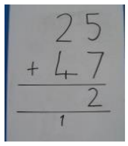
They will now be able to make the links between this representation and the formal column addition when seen alongside each other.

25 + 47

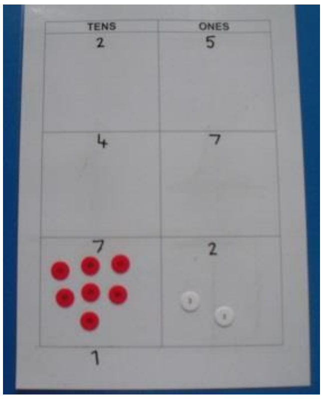


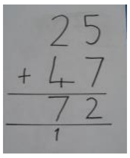






12 ones exchanged to 1 ten and 2 ones





Stage 5	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

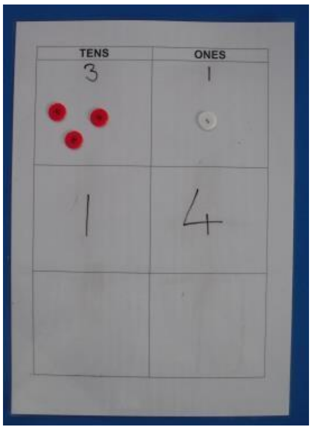
- - - - - - - - Subtraction - - - - - - - -

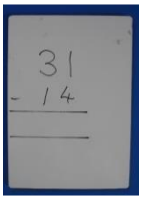
Vocabulary – Ensure the correct vocabulary is used at all stages of learning
subtract, subtraction, take away, minus, decrease, leave, how many are left/left over?, difference between, half, halve, how many more/fewer is.../than...?, how much more/less is...?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

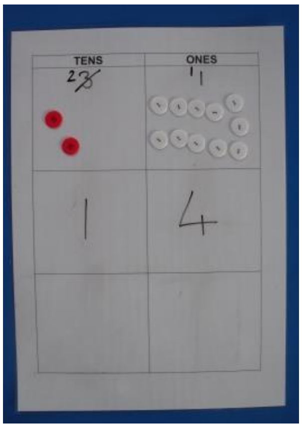
Children are now confident in organising their concrete equipment in a vertical manner for subtraction using the principle of exchange appropriately.

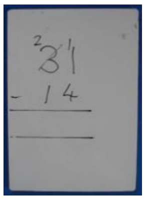
They will now be able to make the links between this representation and the formal column subtraction when seen alongside each other.

31 - 14

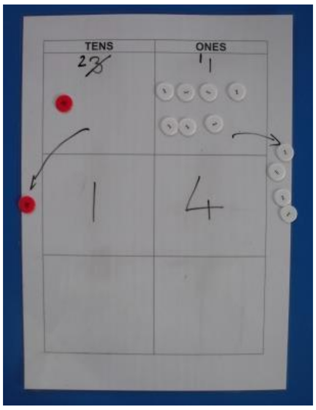




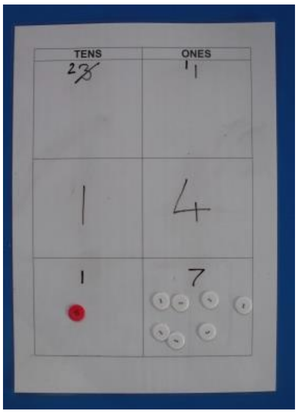


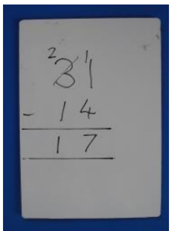


31 is repartitioned into 20 and 11 using the principle of exchange in order to enable us to remove the four ones associated with 14



14 can now be removed from the 31





The remaining equipment can then be slid down to the answer box showing what is left

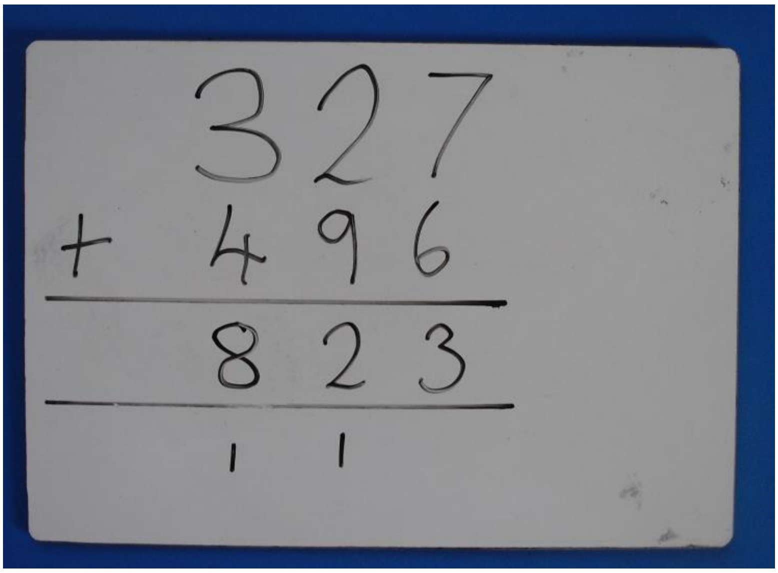
Stage 6	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

+ + + + + + + + Addition + + + + + + + +

Vocabulary – Ensure the correct vocabulary is used at all stages of learning
add, addition, more, plus, increase, sum, total, altogether, double, near double, difference, same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse, how many more to make…?, is the same as

Children will have a full understanding of the links between the concrete representation for column addition and the formal written method.

They will now be able to explore calculating with larger numbers using their understanding of the formal written method.



Calculating with decimals
When working with decimals, the above stages should always be followed for the development of conceptual understanding. The use of concrete equipment is essential at these stages to secure understanding of the value of each digit in a number (e.g. Place Value Counters, Money). Wherever possible, decimal calculations should be linked to real-life experiences, e.g. money and measures.

Stage 6	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

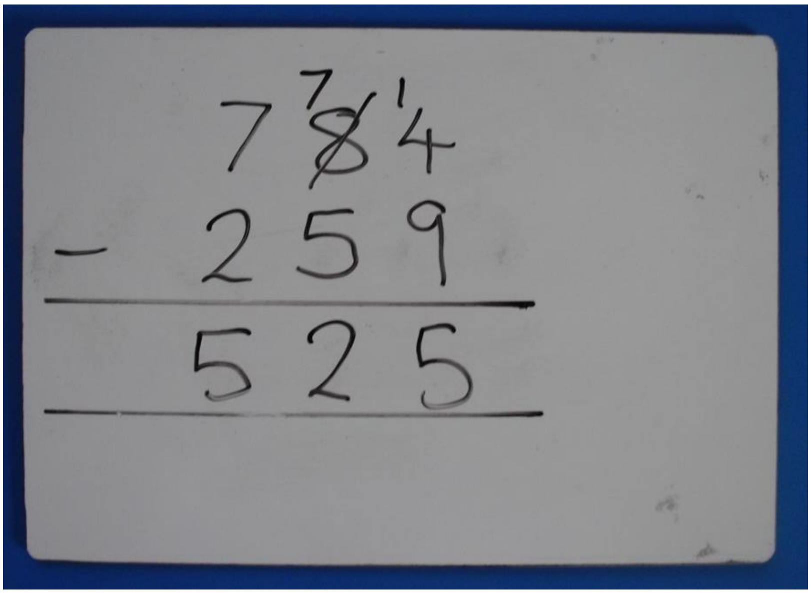
- - - - - - - - Subtraction - - - - - - - -

Vocabulary – Ensure the correct vocabulary is used at all stages of learning

subtract, subtraction, take away, minus, decrease, leave, how many are left/left over?, difference between, half, halve, how many more/fewer is.../than...?, how much more/less is...?, is the same as, equals, sign, tens boundary, hundreds boundary, ones boundary, tenths boundary, inverse

Children will have a full understanding of the links between the concrete representation for column subtraction and the formal written method.

They will now be able to explore calculating with larger numbers using their understanding of the formal written method.



Calculating with decimals

When working with decimals, the above stages should always be followed for the development of conceptual understanding. The use of concrete equipment is essential at these stages to secure understanding of the value of each digit in a number (e.g. Place Value Counters, Money). Wherever possible, decimal calculations should be linked to real-life experiences, e.g. money and measures.

Multiplication

and

Division

×

÷

Stage 1	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

x x x x x x x x Multiplication x x x x x x x x

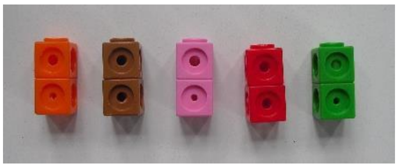
Vocabulary – Ensure the correct vocabulary is used at all stages of learning
counting, steps, each, doubling, scaling, times, twice as big, times as big, count in ones, count in ___, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times…, ten times…, times as (big, long, wide…and so on), repeated addition, array, row, column, double, group in pairs, threes…tens, equal groups of, multiplication, product, inverse

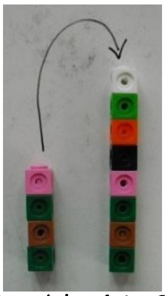
Children will experience practical opportunities involving equal sets or groups using a wide variety of equipment. Practical resources will support children's development of mental images and pictures.

Children will begin to orally count in different multiples including twos, fives and tens making links to natural groupings (e.g. pairs of socks, legs on animals) and the practical resources used.

Children can begin to recognise and continue patterns of multiples using a range of practical resources, e.g. threading beads with two of each colour.

They will begin to use the language and associated representations of doubling.





Double 4 is 8

Stage 1	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷

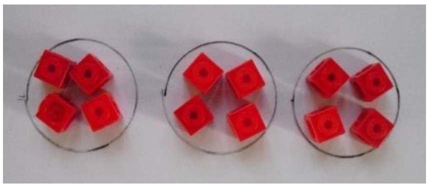
Vocabulary – Ensure the correct vocabulary is used at all stages of learning
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___, ____ equal groups

Children will explore the language of sharing. Children will experience practical activities in 'sharing' objects between a small number of groups/people with the emphasis on sharing equally.

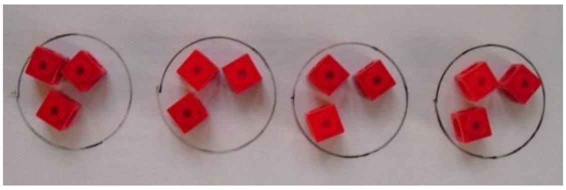
Alongside this, with equal weighting, children should be introduced to 'grouping' objects as a representation of division (e.g. 'each person gets 2') with the emphasis on equal groups.

They will begin to use the language and associated representations of halving.

Children can be encouraged to develop ways of recording their findings using pictures.



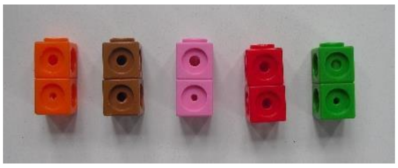
12 shared into 3 equal groups.
12 shared equally into groups of 4.



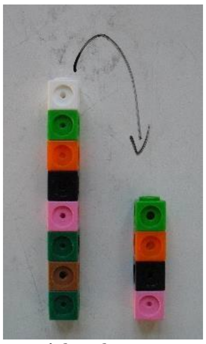
12 shared into 4 equal groups.
12 shared equally into groups of 3.

6 cards shared between 2 people.

6 cards, how many people can have 2 each?







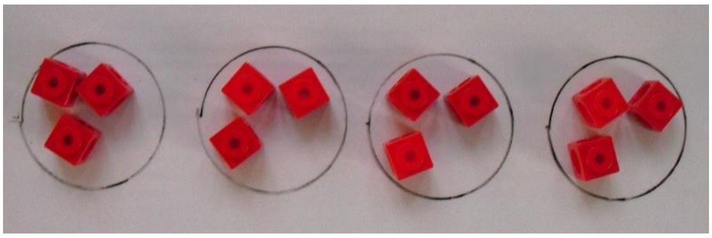
Half of 8 is 4

Stage 2	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

x x x x x x x x Multiplication x x x x x x x x

Vocabulary – Ensure the correct vocabulary is used at all stages of learning
counting, steps, each, doubling, scaling, times, twice as big, times as big, count in ones, count in ___, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times…, ten times…, times as (big, long, wide…and so on), repeated addition, array, row, column, double, group in pairs, threes…tens, equal groups of, multiplication, product, inverse

Children will begin to arrange objects into equal groups to aid counting.



They will continue to count in multiples and begin to relate this to multiplication through finger counting.


"3"


"6"

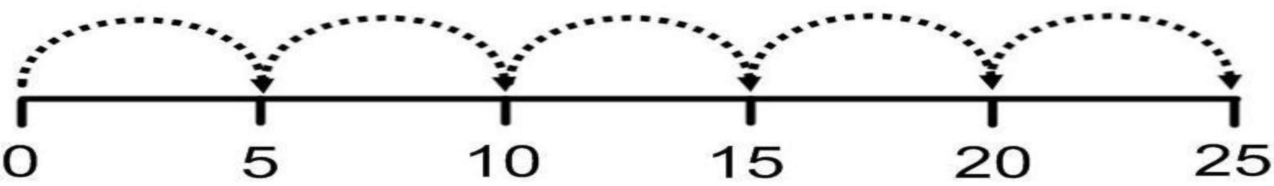

"9"

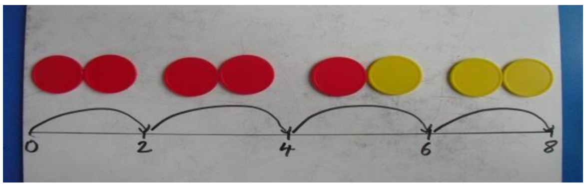

"12"

Children will be introduced to a variety of representations of repeated addition; they will see the representations alongside each other and begin to make connections between them.

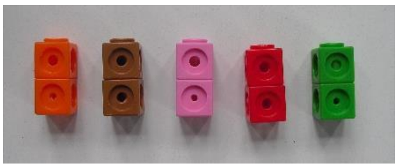








Count in twos from zero

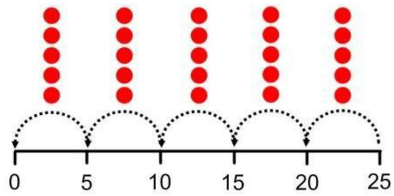


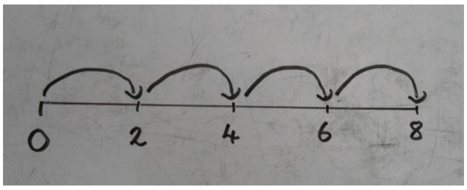
Stage 2	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷

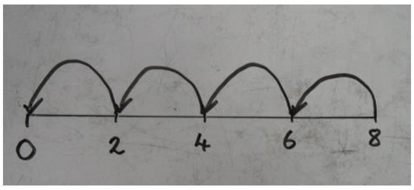
Vocabulary – Ensure the correct vocabulary is used at all stages of learning
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___, ____ equal groups

Children will relate the grouping of objects to repeated subtraction and begin to represent this using a number line whilst continuing to use concrete equipment.





How many 2s are in 8?



How many 2s can we take away from 8?

Children will use their knowledge of counting up in multiples to solve division calculations and recognise that this is the inverse of multiplication.

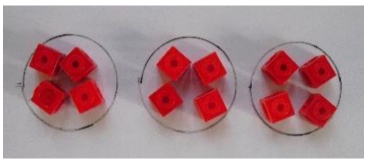
"3"

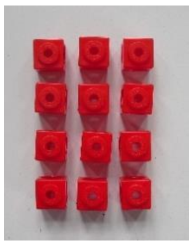
"6"

"9"

"12"

Children will continue to group and share equally using concrete equipment and will now begin to organise their groups into an array rather than scattered groupings.



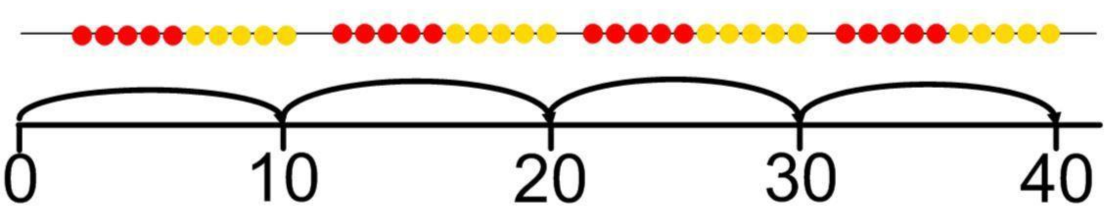


Stage 2 (cont.)	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

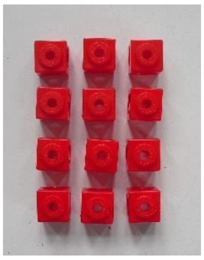
x x x x x x x x Multiplication x x x x x x x x

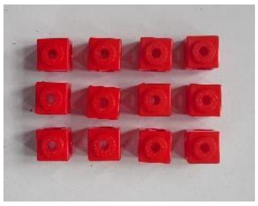
Vocabulary – Ensure the correct vocabulary is used at all stages of learning
counting, steps, each, doubling, scaling, times, twice as big, times as big, count in ones, count in ___, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times…, ten times…, times as (big, long, wide…and so on), repeated addition, array, row, column, double, group in pairs, threes…tens, equal groups of, multiplication, product, inverse

Count in tens from zero



Children will be introduced to the array, using concrete equipment, for small numbers as a way of organising groups to show repeated addition and commutativity. They should explore arrays in the world around us, e.g. egg boxes, baking trays, wrapping papers; and use them to answer the questions such as 'How many eggs would we need to fill the egg box?' 'How do you know?'





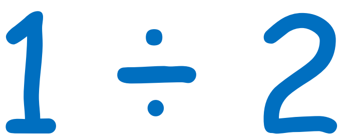
Stage 2 (cont.)	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

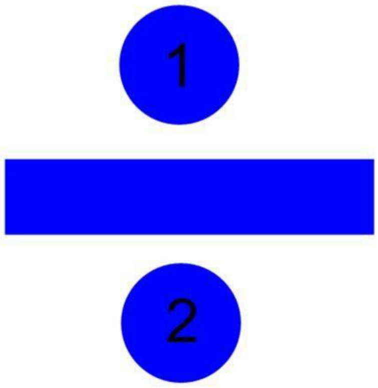
÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷

Vocabulary – Ensure the correct vocabulary is used at all stages of learning
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___, ____ equal groups

The direct link between multiplication and division should be made explicit when using models and representations.

Children will continue to make links between division and fractions. They will be aware that the division sign is the equivalent to the fraction line and so p ÷ q can be written as $\frac{p}{q}$.





Stage 3	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

x x x x x x x x Multiplication x x x x x x x x

Vocabulary – Ensure the correct vocabulary is used at all stages of learning

counting, steps, each, doubling, scaling, times, twice as big, times as big, count in ones, count in ___, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times..., ten times..., times as (big, long, wide...and so on), repeated addition, array, row, column, double, group in pairs, threes...tens, equal groups of, multiplication, product, inverse

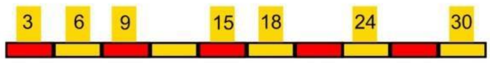
Children will continue to count in multiples and relate this to multiplication through finger counting.


"7"


"14"

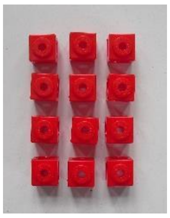

"21"


"28

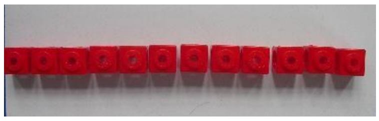


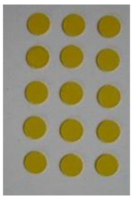
They will be able to model a calculation using a practical array which demonstrated an effective method of counting and the link to repeated addition. Children need to explore related multiplication facts of a given number by making a variety of arrays and explaining what they show.

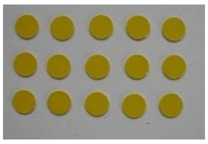
Representing 12


3x4 = 12
4x3 = 12

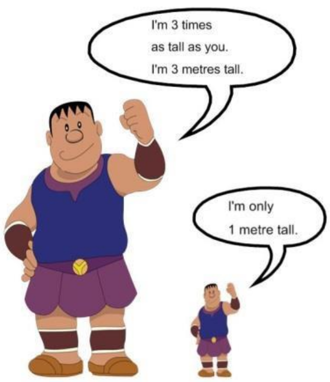

2x6 = 12
6x2 = 12

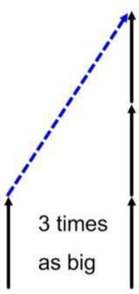

1x12 = 12
12x1 = 12


3+3+3+3+3 = 15


5+5+5 = 15

The children should be confident with their use of the language of scaling when talking about multiplication.





Stage 3	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

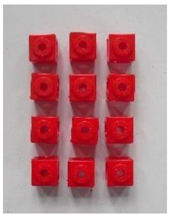
÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷

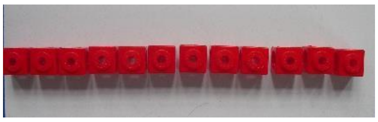
Vocabulary – Ensure the correct vocabulary is used at all stages of learning
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of___, ____ equal groups

Children will continue to use their knowledge of counting in multiples to support the inverse of multiplication and repeated subtraction.

Children will build upon their use of concrete arrays for division recognising the links to repeated subtraction and the inverse of multiplication in order to derive the associated division facts. Children need to explore related division facts of a given number by making a variety of arrays and explaining what they show.

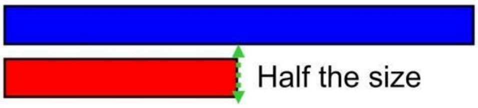
Representing 12

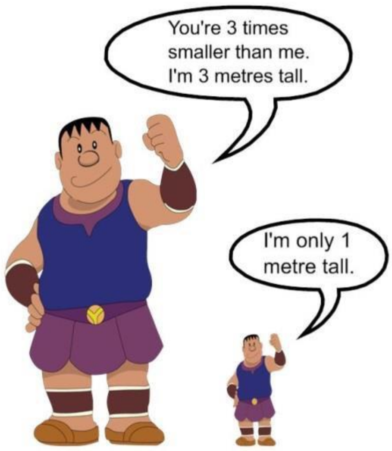




12 into_____equal groups gives_____in each group
12 into equal groups of_____gives_____groups

The children should be confident with their use of the language of scaling when talking about division with links made to simple fractions (e.g. half the size, three times smaller).





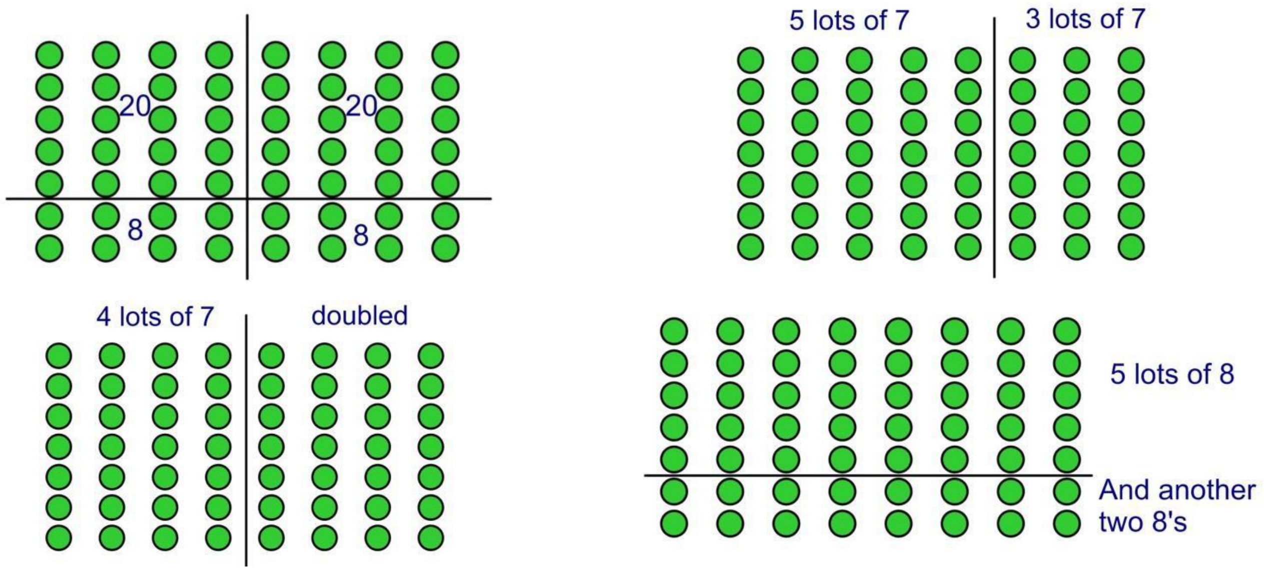
Stage 4	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

x x x x x x x x Multiplication x x x x x x x x

Vocabulary – Ensure the correct vocabulary is used at all stages of learning
counting, steps, each, doubling, scaling, times, twice as big, times as big, count in ones, count in ___, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times…, ten times…, times as (big, long, wide…and so on), repeated addition, array, row, column, double, group in pairs, threes…tens, equal groups of, multiplication, product, inverse

Children will explore practical arrays for larger numbers. They will think flexibly when working with arrays and will be encouraged to look at arrays beyond repeated addition. They will look for 'friendly' numbers to help them efficiently calculate totals within arrays. E.g. for 7x8, children may find counting in 7s or 8s tricky but they can look for 'friendly' numbers which are easier to calculate e.g. 4x5, 4x2, 4x5, 4x2.

Thinking flexibly about 7 x 8



Children should continue to experience the language of scaling (e.g. scaling up pictures by multiplying by powers of 10, multiplying by powers of 1000 in converting between units of measures).

Stage 4

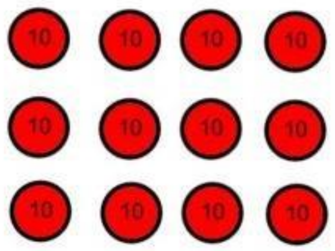
Points to remember
- Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total')
- Use the language '**digit**' not number (number is amount or quantity)

÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷

Vocabulary – Ensure the correct vocabulary is used at all stages of learning
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___, ____ equal groups

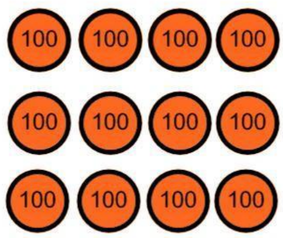
Children will continue to organise groups into an array now working with larger numbers by either grouping or sharing. Children will be able to explain all the facts they know about a given array with no remainder. They should be making arrays with the equipment to establish 'How many in each group?' or 'How many groups?'. Children should continue to experience the language of scaling (e.g. scaling down pictures by dividing by powers of 10, dividing by powers of 1000 in converting between units of measure).

120 ÷ **3**



120 shared equally between 3 is 40.
120 shared equally between 4 is 30.
3 equal groups of 40 make 120.
4 equal groups of 30 make 120.

1200 ÷ **3**



1200 shared equally between 3 is 400.
1200 shared equally between 4 is 300.
3 equal groups of 400 make 1200.
4 equal groups of 300 make 1200.

Stage 5

Points to remember

- Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total')
- Use the language '**digit**' not number (number is amount or quantity)

x x x x x x x x Multiplication x x x x x x x x

Vocabulary – Ensure the correct vocabulary is used at all stages of learning

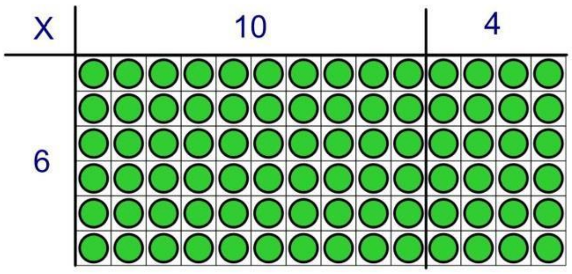
counting, steps, each, doubling, scaling, times, twice as big, times as big, count in ones, count in ___, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times…, ten times…, times as (big, long, wide…and so on), repeated addition, array, row, column, double, group in pairs, threes…tens, equal groups of, multiplication, product, inverse

Children will continue to work with arrays, exploring larger numbers, leading ointo the grid method of multiplication. Practical experiences may still be required for some children as they enter this stage. To begin with, children should see the array with the grid lines. When appropriate, children should move to using the grid displaying the numbers only.

Children should begin using the grid method for 2 and 3 digit by 1 digit numbers and should be given the chance to relate this to facts they know about arrays where needed.

Throughout this stage, children should be encouraged to estimate an approximate answer in order to check for reasonableness and this should become standard practice.

When a child is confident in using the array, the use of a white board should be encouraged to avoid using arrays for 'larger' numbers because this can be very time consuming due to the amount of practical (concrete) equipment required.



(6 x 10) + (6 x 4)
60 + 24
84

X	10	4
6	60	24

Stage 5

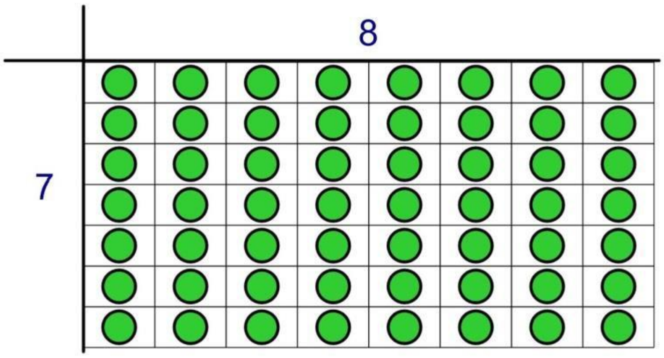
Points to remember

- Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total')
- Use the language '**digit**' not number (number is amount or quantity)

÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷

Vocabulary – Ensure the correct vocabulary is used at all stages of learning
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of ___, ____ equal groups

Children will continue to work with concrete arrays, exploring known multiplication/division facts, with the use of grid lines to begin to make the link to short division where numbers are easily divisible. The children understand that the array within short division can be interpreted for both sharing between, or equal groups of, where the dots within the array represent 1.



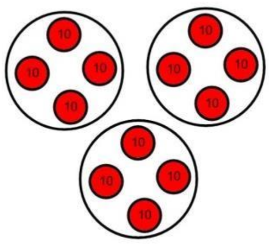
How many equal groups of 7 can I make?

or

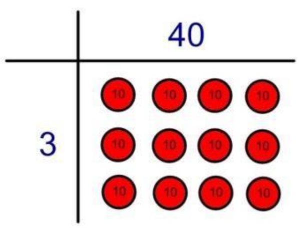
If I put these into 7 equal groups, how many in each group?
(sharing between is represented in the rows)

Children will begin to use counters within an array to show the sharing model of division, using their knowledge of the principle of exchange where necessary. At this stage, children are encouraged to consider the links between the sharing model and fractions.

120 can be exchanged for 12 tens to make an array



120 shared into 3 equal groups gives 40 in each group



We can explicitly see 40 three times; 3 rows of 40, a 1/3 of 120 is 40.
We can divide the array into three parts ad there is 40 in each part.

Stage 6

Points to remember

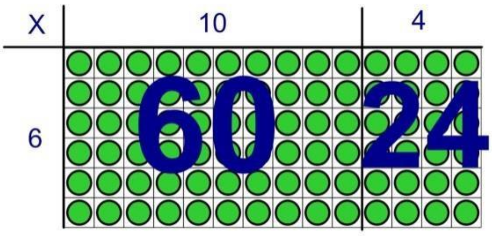
- Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total')
- Use the language '**digit**' not number (number is amount or quantity)

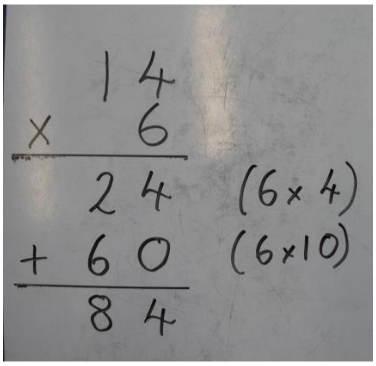
x x x x x x x x Multiplication x x x x x x x x

Vocabulary – Ensure the correct vocabulary is used at all stages of learning

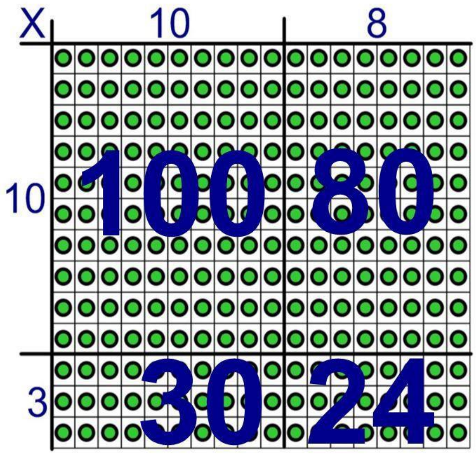
counting, steps, each, doubling, scaling, times, twice as big, times as big, count in ones, count in ___, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times…, ten times…, times as (big, long, wide…and so on), repeated addition, array, row, column, double, group in pairs, threes…tens, equal groups of, multiplication, product, inverse

Children will now be secure in using the grid method for multiplying by one-digit numbers and will begin to explore the links between the grid method and the expanded method of short multiplication.

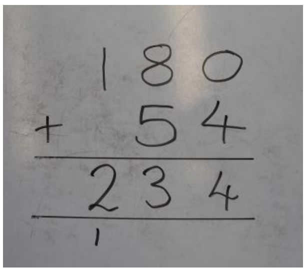




Children will also begin to explore the use of arrays and the grid method for multiplying by two-digit numbers.



X	10	8
10	100	80
3	30	24



Stage 6	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷

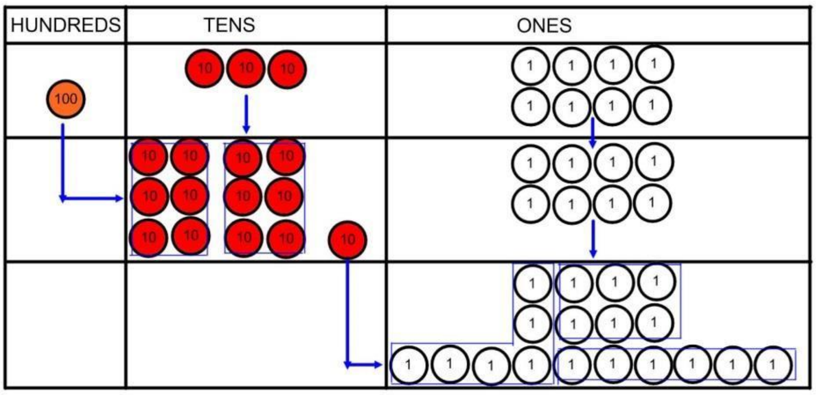
Vocabulary – Ensure the correct vocabulary is used at all stages of learning
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of___,____equal groups

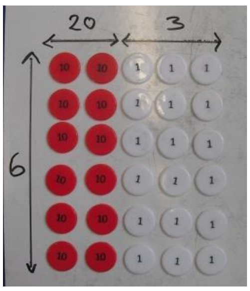
Children will work with equipment to divide any integer by a single digit divisor using their sound knowledge of the principle of exchange.

They will begin to be introduced to numbers that have remainders and will recognise and be able to talk about these when they do not 'fit' into the array.

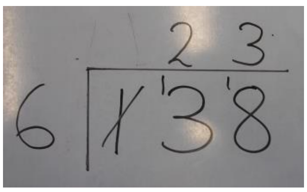
Children will be introduced to the notation of short division, linking with the principle of exchange and how this relates to the practical representations.

Children continue to be encouraged to consider the links between the sharing model and fractions.





In the array, we can explicitly see 23 six times; 6 rows of 23. This is the sharing model.
1/6 of 138 is 23.
We can divide the array up into six equal parts and there is 23 in each part.



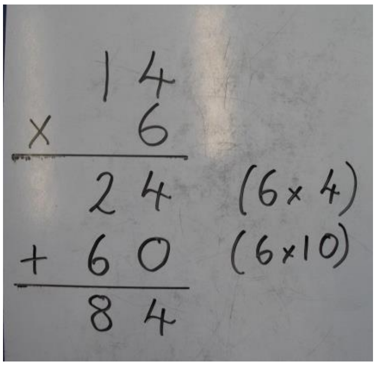
Stage 7

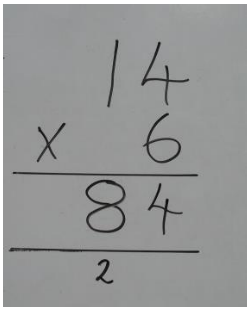
Points to remember
- Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total')
- Use the language '**digit**' not number (number is amount or quantity)

x x x x x x x x Multiplication x x x x x x x x

Vocabulary – Ensure the correct vocabulary is used at all stages of learning
counting, steps, each, doubling, scaling, times, twice as big, times as big, count in ones, count in ___, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times…, ten times…, times as (big, long, wide…and so on), repeated addition, array, row, column, double, group in pairs, threes…tens, equal groups of, multiplication, product, inverse

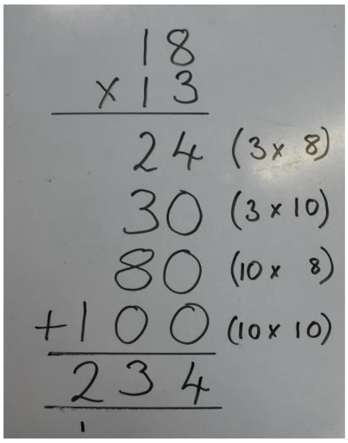
Children will now have a good understanding of the expanded short multiplication method and will begin to represent this as a compact short multiplication for a double digit multiplied by a single digit.





Children will be secure in using the grid method for multiplying by two-digit numbers and will begin to explore the links between the grid method and the expanded method of long multiplication.

X	10	8
10	100	80
3	30	24



Stage 7

Points to remember

- Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total')
- Use the language '**digit**' not number (number is amount or quantity)

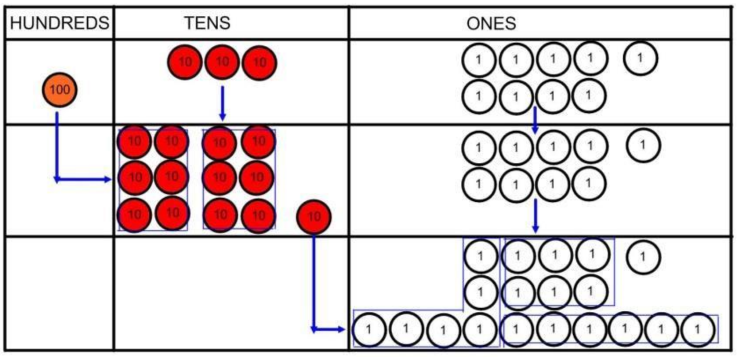
÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷

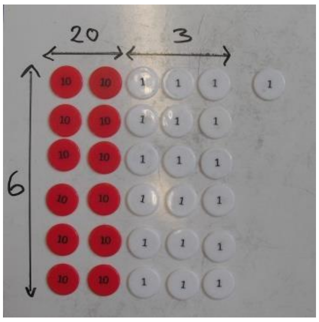
Vocabulary – Ensure the correct vocabulary is used at all stages of learning

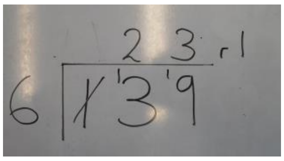
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of___,____equal groups

Children will now be secure in using short division for one-digit divisors with an integer quotient (an answer that is a whole number with no remainder).

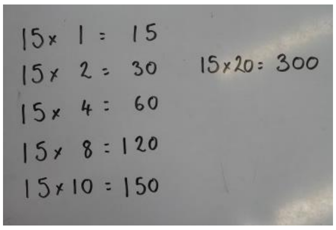
They will now begin to use the short division notation for calculations involving remainders.

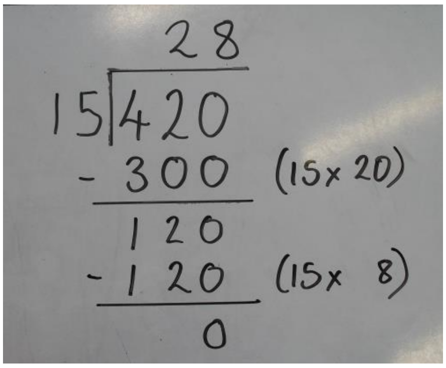






Children will also begin to explore the use of jottings of friendly numbers to support long division of calculations with 2-digit divisors.





Stage 8

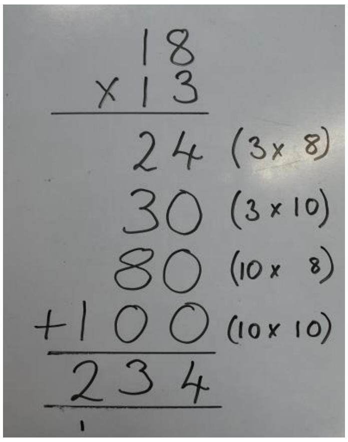
Points to remember
- Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total')
- Use the language '**digit**' not number (number is amount or quantity)

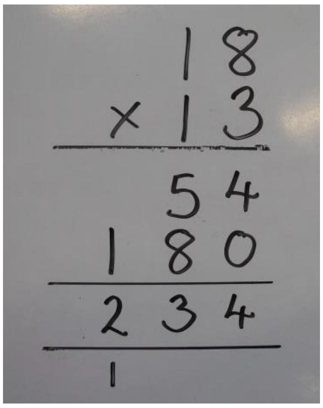
x x x x x x x x Multiplication x x x x x x x x

Vocabulary – Ensure the correct vocabulary is used at all stages of learning
counting, steps, each, doubling, scaling, times, twice as big, times as big, count in ones, count in ___, lots of, groups of, x, times, multiply, multiplied by, multiple of, once, twice, three times..., ten times..., times as (big, long, wide...and so on), repeated addition, array, row, column, double, group in pairs, threes...tens, equal groups of, multiplication, product, inverse

Children will now have a good understanding of the short multiplication method.

Children will now have a good understanding of the expanded long multiplication method and will begin to represent this as compact long multiplication.





Calculating with decimals
When working with decimals, the above stages should always be followed for the development of conceptual understanding. The use of concrete equipment is essential at these stages to secure understanding of the value of each digit in a number (e.g. Place Value Counters, Money). Wherever possible, decimal calculations should be linked to real-life experiences, e.g. money and measures.

Stage 8	Points to remember • Use the language '**calculation**' not 'sum' ('sum means 'plus' or 'total') • Use the language '**digit**' not number (number is amount or quantity)

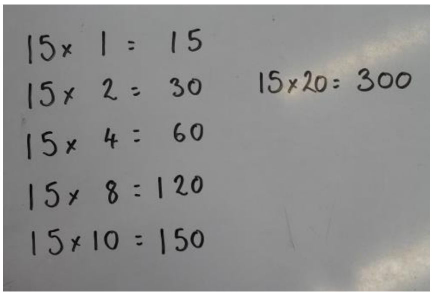
÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷ Division ÷ ÷ ÷ ÷ ÷ ÷ ÷ ÷

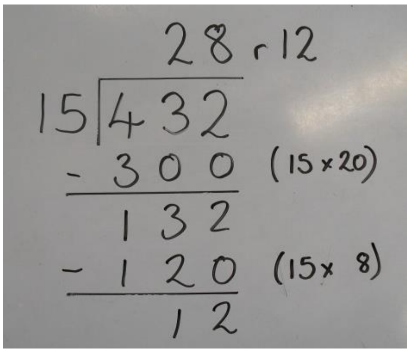
Vocabulary – Ensure the correct vocabulary is used at all stages of learning
halve, share, share equally, one each, two each, three each…, divide, division, divided by, divided into, left, left over, remainder, quotient, divisible by, inverse, exchange, repartition, divisor, scaling, repeated subtraction, array, row, column, equal groups of___,____equal groups

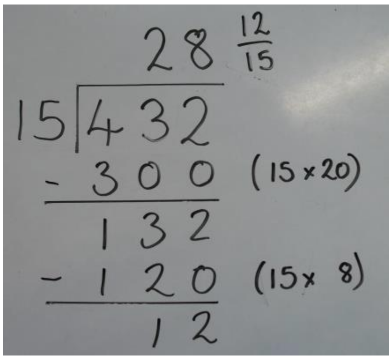
Children will now be secure in using short division for one-digit divisors and long division for two-digit divisors with an integer quotient.

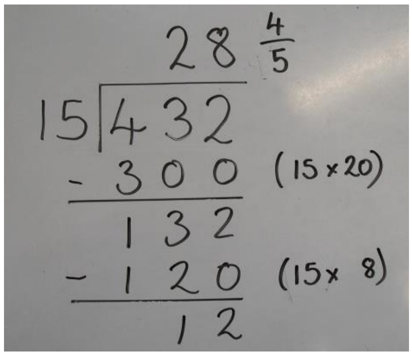
They will now explore the use of long division for two-digit divisors which may include a remainder.

The children will begin to interpret remainders as whole number remainders, fractions or by rounding, as appropriate for the content.









Calculating with decimals

When working with decimals, the above stages should always be followed for the development of conceptual understanding. The use of concrete equipment is essential at these stages to secure understanding of the value of each digit in a number (e.g. Place Value Counters, Money). Wherever possible, decimal calculations should be linked to real-life experiences, e.g. money and measures.